

Name: _____

Please answer any 6 questions, showing all calculations - 25 points each, 150 total.

Particle in a box: $\Psi_n = (2/a)^{1/2} \sin(2n\pi x/a)$ $E = h^2 n^2 / 8m a^2$.

1. Which of the following wavefunctions are orthogonal to $\Psi_1 = A_1 \cos x$?

$$\Psi_2 = A_2 \exp(-x^2)$$

$$\Psi_3 = A_3 \sin 3x/2$$

2. What are the most probable locations to find an electron in a 2.0 Å box in the $n = 5$ PIB state? What is the expectation value for the location of this electron?
3. What is the probability of finding an electron within 1.0 Å of the center of the first excited state ($n = 2$) for a particle in a 5.0 Å box?
4. Show that $f = 10 e^{-8x}$ is an eigenfunction of the operator d/dx . What is the eigenvalue?
5. If the ground state of a harmonic oscillator is $\Psi_1 = A \exp(-k x^2)$, calculate the 2nd excited state given that the raising operator is given by $\mathbf{a}^\dagger \Psi_n = (\hbar / m \omega)^{1/2} \Psi_{n+1}$ and that $\mathbf{a}^\dagger = (1/2)^{1/2} [k^{1/2} \mathbf{x} - i \mathbf{p} / m^{1/2}]$.
6. Graphically draw the approximate wavefunctions and energy level spacing for the first 3 levels of a potential energy of $V(-Z < x < Z) = k x^2$ and $V(x > Z \text{ or } x < -Z) = \infty$.
7. Identify the following statements as true or false:
A) Light exhibits both wave-like and particle-like properties.
B) Several different energy eigenfunctions can correspond to the same eigenvalue.
C) Several different eigenvalues can correspond to the same eigenfunction.
D) The uncertainty in the position of a particle can vanish if nothing is known about the momentum.
E) Measurement of the energy of a particle in a 1D box determines the quantum state of the system.
F) Measurement of the energy of a particle in a 3D box determines the quantum state of the system.
8. A sodium lamp emits yellow light (550 nm). How many photons does it emit each second if its power is (a) 1.0 W, (b) 100.0 W?
9. Show that the particle on a ring wavefunctions, $\Psi_m = (1/2\pi)^{1/2} \exp(i m \theta)$, with different quantum numbers are orthogonal.
10. Calculate the value of the commutator $[\mathbf{L}_x, \mathbf{L}_y]$ using $\mathbf{L}_x = \mathbf{y} \mathbf{p}_z - \mathbf{z} \mathbf{p}_y$, $\mathbf{L}_y = \mathbf{z} \mathbf{p}_x - \mathbf{x} \mathbf{p}_z$, $\mathbf{L}_z = \mathbf{x} \mathbf{p}_y - \mathbf{y} \mathbf{p}_x$, and $[\mathbf{x}, \mathbf{p}_x] = [\mathbf{y}, \mathbf{p}_y] = [\mathbf{z}, \mathbf{p}_z] = i \hbar$.
11. Describe in your own words how the black body experiment required physicists and chemists to re-think classical physics and arrive at the new quantum model proposed by Planck.