Examination 1
Chemistry 262
Name: __________________________

1. What, specifically, leads to quantization of energy levels in the harmonic oscillator problem? (15 points)

2. For each of the two functions given, determine whether it is an eigenfunction of the momentum operator and its eigenvalue if it is? (20 points)

\[ \hat{\rho}_x = \frac{\hbar}{i} \frac{d}{dx} \]

\[ \psi_1 = A \exp(-ikx) \]

\[ \psi_2 = Ax \exp(-kx^2) + B \exp(-kx^2) \]
3. For each of the three functions below explain why it can or cannot be a real world wavefunction. (15 points)

\[ \psi_1 = \frac{A}{x} \exp(-ix) \quad \psi_2 = A|x| \quad \psi_3 = \begin{cases} A \exp(x) & x \leq 0 \\ 2A \exp(-ikx) & x \geq 0 \end{cases} \]

4. How would you calculate the mean square position expectation value for the second excited state particle in a box. (20 points, 5 Bonus points if you perform integration using the integral provided)

\[ \psi_n = \frac{1}{\sqrt{L}} \cos \frac{n\pi x}{2L} \quad \int x^2 \cos^2 x \, dx = \frac{x^3}{6} + \frac{2x^2 - 1}{8} \sin2x + \frac{x \cos2x}{4} + C \]
5. What is the energy for a particle with the wavefunction below in a one dimensional harmonic oscillator potential. (5 points for setting up derivatives, 5 points for actually performing the correct differentiation, 5 Bonus points for setting the additional calculation of the mean square momentum of this state)

\[ \psi_1 = (2x^2 - 1) \exp(-x^2/2) \quad \hbar^2/m = 2 \quad \hat{H} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{\hbar^2 x^2}{2m} = -\frac{d^2}{dx^2} + x^2 \]
6. Draw pictorially the wavefunctions for particles with the energy levels indicated below (E₁ and E₂) for the following potential. Notice that energy state E₁ is less than the middle potential V₂ but larger than the other two potentials V₁ and V₃. Energy state E₂ on the other hand is larger than V₁, V₂ and V₃. Indicate where the wavelength of the wavefunction will be largest and smallest. Also, indicate where the wavefunction has a sinusoidal form and where it has an exponential form. (20 points)