Please answer 5 of the 6 Questions

1. Describe one example where the quantization of the energy of light was used to give a more accurate or complete mathematical description of a physical process. Be specific about the reasons why the quantization plays an important role and why classical models failed. (15 points)

2. A two-dimensional harmonic oscillator has the potential energy function

   \[ V(x,y) = \frac{k}{2}(x^2 + y^2). \]

   Write the time-independent Schrödinger equation and describe its solutions in terms of Dirac notation kets. Note you would need to invoke separation of variables here. What would be the energy eigenvalues and the degeneracies for the first 10 energy levels. (15 points)

3. Draw sketches of the second and third wavefunctions for the particle in a box of length L. Without doing the integral mathematically, describe graphically why these are indeed orthogonal wavefunctions. Do the same for the first and second wavefunctions of a harmonic oscillator. (15 points)

4. Find \( \langle E \rangle \) for a non-equilibrium harmonic oscillator wavefunction given below.

   \[ \psi = \sqrt{\frac{3}{2}} \psi_0 + \sqrt{\frac{3}{2}} \psi_1 + \sqrt{\frac{3}{2}} \psi_2 \]

   Note you need to do an expectation value calculation using operators as we have done before and the \( \psi_n \) are the \( n \)th solutions to the harmonic oscillator problem. Remember orthogonal rules. Second, calculate the standard deviations, \( \sigma_E \), you would observe in this sort of measurement. This is equivalent to calculating \( \sigma_E = (\langle E^2 \rangle - \langle E \rangle^2)^{1/2} \). (15 points)

5. Calculate the wavelength of the photon emitted when a 1.0x10\(^{-27}\) g particle in a box of length 6.0 Å goes from the \( n = 5 \) to the \( n = 4 \) state. (15 points)

6. A 100-W sodium-vapor lamp emits yellow light of wavelength 590 nm. Calculate the number of photons emitted per second. (15 points)