Fundamental Constants

- Speed of light, \( c = 2.997925 \times 10^8 \) m s\(^{-1}\)
- Pi, \( \pi = 3.14159 \)
- Planck’s constant, \( h = 6.62618 \times 10^{-34} \) J s
- Boltzmann’s constant, \( k = 1.38066 \times 10^{-23} \) J K\(^{-1}\)
- Atomic mass unit, \( amu = 1.66056 \times 10^{-27} \) kg
- \( e^x = 1 + x + x^2/2 + (x << 1) \)

1. Given the black body radiation formula below and the knowledge that the peak in the sun’s energy spectrum occurs at 480 nm, estimate the exterior temperature of the sun. For full credit demonstrate how the black body radiation formula leads to Wein’s law (the final step of this proof requires that \( e^{hc/kT} - 1 \) be approximated by \( e^{hc/kT} \)).

\[
U = \frac{8\pi hc}{\lambda^5} \left( \frac{1}{e^{hc/kT} - 1} \right)
\]

(25 points)

2. The ground-state wavefunction the hydrogen atom has the form \( \psi (r) = Ne^{-ar} \). Normalize this spherically symmetrical function. The volume element is \( d\tau = \sin \theta d\theta d\phi r^2 dr \), with \( 0 \leq \theta \leq \pi \), \( 0 \leq \phi \leq 2\pi \), \( 0 \leq r \leq \infty \). The final integral will require you to integrate by parts.

\[
\int udv = uv - \int v du
\]

(25 points)

3. Write the wavefunction for a particle of mass 1.00 g travelling from \( x = -\infty \) to \( x = +\infty \) at a speed of 10.0 m s\(^{-1}\) along a line with \( V = 0 \) everywhere.

(25 points)

4. Write down the Hamiltonian for a particle freely rotating about a point in two dimensions. Derive both the general quantized eigenfunction for this Hamiltonian and the energy of this eigenfunction. In addition, show the correct normalization of this wavefunction (remember \( 0 \leq \phi \leq 2\pi \)).

(25 points)

5. Write out all of the allowed (using a table of microstates) term symbols for the electronic configuration \( p^3 \).

(25 points)

6. Compute the relative populations of the lowest five rotational energy levels for the molecule \(^{12}\text{C}^{16}\text{O}\) where the mass of \(^{12}\text{C}\) is 12.000000 amu, \(^{16}\text{O}\) is 15.994915 amu and the ground state bond length is 113.07 pm.

(25 points)
7. Describe the basic differences (experimental setup, selection rules, spectrum appearance, example molecules, etc.) between normal absorption rotational spectroscopy and Raman rotational spectroscopy. (25 points)

8. Calculate the $x$ - momentum ($p_x$) eigenvalue of each of the following functions (if it is not an eigenfunction, state this as well). (25 points)

\[
\begin{align*}
    f_1 &= ax \\
    f_2 &= \cos(ax) \\
    f_3 &= \exp(-i ax) \\
    f_4 &= \exp(-a x^2)
\end{align*}
\]

9. Describe the differences (shape of potential well, energy level locations, wavefunctions, expectation values, etc.) between the a harmonic oscillator model and the true potential for a vibrating diatomic molecule. (25 points)